Cryptography for Next Generation TLS:
Implementing the RFC 7748 Elliptic Curve448 Cryptosystem in Hardware

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ABSTRACT
With RFC 7748 the two elliptic curves Curve25519 and Curve448 were proposed for the next generation of TLS. Both curves were designed and optimized purely for software implementation; their implementation in hardware or physical protection against side-channel attacks were not considered in the design phase. Recently, it has been shown that for Curve25519 an efficient implementation in hardware along with side-channel protection is feasible – yet results for the high-security Curve448 are missing. In this work we demonstrate that Curve448 can indeed be efficiently and securely implemented in hardware. We present a novel architecture for Curve448 that can compute more than 1000 point multiplications per second with 1580 logic slices and 33 DSP units of a Xilinx XC7Z020 FPGA.

Keywords
Curve448, RFC 7748, FPGA, high-performance, implementation, side-channel, countermeasure

1. INTRODUCTION
Elliptic Curve Cryptography (ECC) has become the most predominant scheme in the area of asymmetric cryptography during the last decade. However, recent revelations on manipulations and backdoors have undermined the confidence in existing schemes and led to discussion among researchers and standardization bodies on the selection of new curves and the rigidity of existing curve generation processes. Within these discussions, the Internet Engineering Task Force (IETF) Transport Layer Security (TLS) working group requested new recommendations on elliptic curves for the next generation of TLS on which the Internet Research Task Force (IRTF) Crypto Forum Research Group (CFRG) has selected two candidates for the RFC 7748: Curve25519 and Curve448. Simultaneously, the IETF cardle group proposed both curves for application in future protocols such as DNSSEC. With further emergence of, e.g., self-driving cars, high-performance TLS (including Curve25519 and Curve448) has to be implemented on various physical and embedded devices. But since both curves were primarily designed for software platforms (x86/x64) and without explicit evaluation of their resistance against low-level physical such as side-channel attacks, we still need to investigate the hardware implementation of both curves in combination with suitable countermeasures.

Contribution. In this work, we present an efficient hardware implementation of Curve448 for modern Xilinx Field-Programmable Gate Array (FPGA) devices. Our work fills the gap left from the previous work on Curve25519 [14, 15]. Despite of the similarity of Curve448 and Curve25519, we were faced with completely revising the design rationales due to specially-crafted parameters and the significantly increased field size of 448 bits. Interestingly, we can demonstrate that even in light of the asymptotically growing complexity in the field size, Curve448 still can provide similarly high throughput on a mid-range Xilinx XC7Z020 FPGA, providing a performance of more than 1000 point multiplications per second at moderate resource costs.

Outline. This work is organized as follows: In Section 2 we summarize and discuss previous but related work of relevance before we give a brief introduction into the topic of ECC in Section 3, focusing on Curve448 in particular. Before presenting the final implementation details in Section 5, we discuss optimization strategies and justify our final design choices in Section 4. Eventually, implementation and performance results are summarized and presented in Section 6 before we conclude in Section 7.

2. PREVIOUS WORK
ECC has been proposed independently by Neal Koblitz [7] and Victor Miller [10] in 1985. Since then, it has established as a viable field of research in public-key cryptography and has been implemented in a great number of industrial products. Up to date, there exists a plethora of publications on hardware implementations that we will not cover or discuss within the scope of this work. Instead we restrict ourselves to briefly list the most relevant ones. However, for a detailed history and list of references we would like to refer the interested reader to [2].

As one of the first ECC architectures implemented on a FPGA, in 2001, Orlando and Paar [12] presented a design using pre-computations and Montgomery multiplications for improved performance results. Their work was followed by many more, including designs based on dedicated multi-

3. BACKGROUND

In this section we will briefly cover basic aspects of ECC. Note, however, that we will only focus on ECC over prime fields. Further, we give a short introduction to Curve448 and its specification.

3.1 Elliptic Curve Cryptography

Given an elliptic curve $\mathcal{E}$ over the Galois Field $GF(p)$ (with $p > 3$), defined by the Edwards equation

$$\mathcal{E}_d : y^2 + x^2 = 1 + dx^2 y^2 \pmod{p}, \quad (1)$$

we can define tuples of affine coordinates $(x_i, y_i)$ as points $P_i \in \mathcal{E}$ on the elliptic curve. Considering two points $P_i, P_j$, we can define the basic group operations as point addition if $i \neq j$ and point doubling if $i = j$. Moving from affine coordinate to projective coordinate representations, hence defining points on $\mathcal{E}$ as tuples $(x_i, y_i, z_i)$, curve equations can be relaxed in order to reduce the number of costly operations (i.e., modular inversions).

Applying elliptic curves for cryptographic purposes relies on the Elliptic Curve Discrete Logarithm Problem (ECDLP) which states that, given two points $P, Q \in \mathcal{E}$, it is hard to find a scalar $k$ such that $Q = k \cdot P$ holds, whereas the point multiplication is defined as a k-fold point addition of $P$.

Eventually, the ECDLP is the fundamental problem that allows building cryptographic protocols and schemes based on elliptic curves. Common examples include the Elliptic Curve Diffie-Hellman (ECDH) key exchange, the Elliptic Curve Digital Signature Algorithm (ECDSA) and the ElGamal encryption scheme.

3.2 Specification of Curve448

Ed448-Goldilocks, in RFC 7748 denoted as Curve448, was proposed and designed by Mike Hamburg [5] in order to provide a high-security alternative (i.e., a field size between 384 and 521 bits) to existing NIST [9] and Brainpool curves [8]. The design process of Curve448 strictly heeded the SafeCurves policies and criteria [1] which should enable simple but secure implementations of an elliptic curve that meets all requirements.

3.2.1 Field Arithmetic

Curve448$^1$ is an untwisted Edwards curve defined by Equation 1, with a security level of 224 bits and factor $d = -390.81$, the Solinas prime $p = 2^{448} - 2^{224} - 1$ obtained from its golden ratio $\phi = 2^{224}$. Technically, Curve448 processes 448-bit values over $GF(p)$ but still is highly versatile and flexible for software implementations and different platforms (ranging from 8- to 64-bit) due to the fact that $448 = 56 \times 8 = 28 \times 16 = 14 \times 32 = 7 \times 64$.

Due to its golden ratio $\phi$ the Solinas prime allows fast and efficient Karatsuba-based multiplication of two operands $A = (a_0 + a_1 \phi)$ and $B = (b_0 + b_1 \phi)$, such that:

$$C = A \cdot B = (a_0 + a_1 \phi) \cdot (b_0 + b_1 \phi) = (a_0 b_0 + a_1 b_1) + ((a_0 + a_1) \cdot (b_0 + b_1) - a_0 b_0) \phi \quad (2)$$

Eventually, modular inversion can be tweaked compared to modular inversions based on Fermat’s Little Theorem (FLT) using that, if $p \equiv 3 \pmod{4}$, the Inverse Square Root (ISR) can be computed as $1/p = x^{(p-3)/2}$, which directly leads to:

$$x^{-1} = x \cdot \left(\frac{1}{x^2}\right)^{\frac{p-3}{4}} = x \cdot \left(\frac{x}{x^2}\right)^{\frac{p-3}{4}} = x^{p-2} \quad (3)$$

3.2.2 Group Arithmetic

According to RFC 7748, the function X448 performs a variable scalar-point multiplication on the Montgomery curve (Curve448) using the Montgomery ladder algorithm [11] that enables the computation of a point addition and a point doubling in a single combined step. Hence, given two points $Q_1, Q_2 \in \mathcal{E}$ and their difference $Q_3 = Q_1 - Q_2$, a single step of the Montgomery ladder algorithm computes two points $Q_4, Q_5 \in \mathcal{E}$ such that $Q_4 = 2 \cdot Q_1 = (x_4, z_4)$ is the point doubling with:

$$x_4 = (x_1 - z_1)^2 \cdot (x_1 + z_1)^2 \quad (4)$$

$$z_4 = 4x_1z_1 \cdot (x_1^2 + dx_1z_1 + z_1^2) \quad (5)$$

and $Q_5 = Q_1 + Q_2 = (x_5, z_5)$ is the point addition with:

$$x_5 = z_3((x_1 - z_1) \cdot (x_2 + z_2) + (x_1 + z_1) \cdot (x_2 - z_2))^2 \quad (6)$$

$$z_5 = x_3((x_1 - z_1) \cdot (x_2 + z_2) - (x_1 + z_1) \cdot (x_2 - z_2))^2 \quad (7)$$

Fortunately, these equations are independent of the y-coordinate which can be omitted during the intermediate computation and only has to be restored for the final result. In total, a single step of the Montgomery ladder algorithm involves 6 multiplications, 4 squarings, 4 additions, and 4 subtractions over $GF(p)$.

3.2.3 Point Multiplication

Given a public point $P \in \mathcal{E}$ and a secret 448-bit scalar $k$, the point multiplication routine computes another point $Q = k \cdot P$ using a sequence of consecutive Montgomery ladder steps. For this purpose, the scalar is scanned bitwise (starting from the most significant bit) and depending on the value of the currently processed bit, inputs to a single step of the Montgomery ladder algorithm are swapped. Hence, starting from the base point $P$, the point at infinity $\mathcal{O}$ and their difference (i.e., $P$), the point multiplication finally yields the resulting point $Q$ after the execution of 448 steps.

3.3 Side-Channel Protection

Side-Channel Analysis (SCA) is a common attack to exploit information leakage related to internals of physical devices e.g., by inspecting and analyzing power consumption, electromagnetic radiations, or timing behavior of the device, in order to extract secret information. Hence, in order to prevent SCA attacks, modern cryptographic implementations usually encompass protection and countermeasures against various physical attacks. Fortunately, the application of the Montgomery ladder algorithm inherently provides...
basic protection against timing and Simple Power Analysis (SPA). However, dedicated countermeasures against more sophisticated attacks such as Differential Power Analysis (DPA) still need to be addressed. Since there is a wealth of different countermeasures against DPA attacks, we refer the interested reader to an overview of Fun et al. [3], but only introduce point randomization and scalar blinding as an example.

3.3.1 Point Randomization

SCA adversaries have physical access to cryptographic implementations and can at least observe inputs and outputs to the device including additional side-channel information. In order to prevent any extraction of cryptographic secrets using statistical analysis of the side-channel information, common countermeasures try to randomize the process data and the side-channel leakage. Point randomization takes advantage of the additional degree of freedom introduced by the application of projective coordinate representations in order to randomize the representation of a base point using a randomly chosen $\lambda$. During the base point transformation process (from affine to projective coordinates), the random factor is applied as shown in Equation 8, which yields in different point representations (depending on $\lambda$).

$$P_r = (x_{P_r}, z_{P_r}) = (\lambda x_P, \lambda z_P) = (\lambda x_P, \lambda)$$

Fortunately, point randomization does not affect the correctness of the underlying scheme, as proven in Equation 9.

$$x_{P_r} \cdot z_{P_r}^{-1} = \lambda x_P \cdot \lambda^{-1} = x_P \pmod{p}$$

3.3.2 Scalar Blinding

Instead of randomizing the base point $P$, scalar blinding deals with the randomization of the second input parameter of point multiplication operation that computes $Q = k \cdot P$. Again, choosing a random factor $r$, the original scalar $k$ is blinded with a multiple of the group order $|E|$, as shown in Equation 10.

$$k_r = k + r \cdot |E|$$

Given the blinded scalar $k_r$, correctness of the underlying scheme is given by the fact that the product of group order and base point returns the point at infinity. Hence, the point multiplication still results in $Q$ as shown in Equation 11.

$$k_r \cdot P = (k + r \cdot |E|) \cdot P = k \cdot P + r \cdot O = k \cdot P$$

4. DESIGN RATIONALES

Hardware implementations of modern cryptographic primitives can be designed in many ways and optimized from different perspectives, such as performance and throughput, resource utilization and hardware footprint, physical and side-channel security and many more. In this section we discuss several optimization strategies and different solution approaches in order to achieve the chosen objectives for our target implementation.

4.1 Implementation Objectives

To the best of our knowledge, there are hardly any published results on hardware implementations targeting ECC beyond the security level of 128 bits. Since the main target for this work is to investigate the suitability of Curve448 in hardware, we designed our architecture to achieve an optimal trade-off in terms of primarily maximizing the performance and throughput while exploring opportunities for resource cost optimization as secondary goal.

4.2 Optimization Objectives

In order to increase the performance of our design to a maximum we applied several optimization strategies which we list in the following.

Register Balancing. It is a common approach to use additional register stages to minimize the critical path delay of hardware architectures. In particular for FPGA implementations this is straightforward since registers are placed after every logic element and would be wasted if they remain unused. We applied manual register balancing to minimize the critical path of each DSP core by using internal register stages in consideration of the latency added by each individual stage.

Parallelization. In particular, hardware implementations can benefit from parallelization techniques which can be applied on various levels of the design hierarchy. We put a particular focus on the parallelization of all required base field operations (addition, subtraction, multiplication) by extensive use of DSP blocks. The placement of multiple core instances on the same device is another opportunity to increase performance, only limited by the available resources of the physical device (simple scalability).

Resource Sharing. Naturally, faster designs and better performance can be purchased by spending more resources. Yet resource sharing may allow reusing existing components instead of implementing additional ones. Similar to parallelization, resource sharing can be applied at different levels of the design hierarchy, i.e., within a single but also among several instances (multi-core design).

Data Path Expansion. For efficient parallel computation, the internal data path of the design has to be expanded in order to provide each component without pipeline stalls. Since this naturally will increase the area utilization of the architecture, we investigated a best trade-off between either a serial and low-area or a parallel and low-latency design.

In general, critical path minimization and parallelization are common approaches to increase the throughput of hardware architectures. We explored several combinations and techniques to achieve the maximum frequency while maintaining a balanced resource utilization in order to allow the implementation of multiple instances in parallel (multi-core design). In the following, we provide a synopsis on our findings and optimization strategies.

4.3 Field Arithmetic Operations

According to Section 3.2.1, all operations on elliptic curves over $GF(p)$ are based on operations over finite fields, such as modular addition, subtraction, multiplication, and inversion. Hence, the fundamental component of our hardware architecture is the Field Arithmetic Unit (FAU) which has...
to be designed and optimized thoroughly (cf. Section 5.1) since its operations have a great impact on the overall performance. In this context, we finally achieved better results by entirely manual optimization instead of relying on automated tools. In the following, we explain and discuss all relevant design choices for our Field Arithmetic Unit that is optimized for use with Curve448.

4.3.1 Modular Inversion

In general, hardware designers have to decide whether to perform modular inversion using dedicated units, e.g., based on the binary Extended Euclidean Algorithm (EEA), or to reuse modular multiplication to compute the inversion using FLT or the ISR tweak. We decided against a dedicated unit due to its little performance gain with respect to its area costs (based on a single core architecture). Instead, we decided to perform modular inversion using the ISR tweak as given in Equation 3.

4.3.2 Modular Multiplication

For modular multiplication, we investigated and compared two different approaches: Karatsuba Multiplication and Schoolbook Multiplication with Interleaved Reduction. Since the primary goal of this work is to explore the suitability and performance of the point multiplication for Curve448, we decided to build a modular multiplication unit based on the Schoolbook Multiplication with Interleaved Reduction. This approach has a lower latency compared to the Karatsuba Multiplication in hardware and most important, it maps more naturally to the available hardware resources (DSP units).

4.3.3 Architectural Customizations

In Section 4.2 we presented a general set of optimization strategies that we considered for the entire design process.

Based on this, we applied manual register balancing to minimize the critical path of each DSP core by using internal register stages in consideration of the latency added by each individual stage. Further, registers are balanced between our Arithmetic Core (AC) and Reduction Core (RC) in the same manner.

5. IMPLEMENTATION DETAILS

In this section, we provide implementation details of our Curve448 hardware architecture, in particular the arithmetic units of our design. In addition, we provide details on an enhanced version of our basic architecture which provides further protection mechanism and countermeasures against side-channel attacks. Most of all, we like to point out that this additional protection can be integrated into the basic architecture with limited effort and overhead.

5.1 Field Arithmetic Unit

As mentioned earlier, the FAU (as sketched in Figure 2) is the basic component of our architecture. In principle, it can be subdivided into two different components, an AC and a RC, that we explain in detail in the following.

Arithmetic Core (AC). In order to perform arithmetic operations, the body of our AC is implemented using 28 fully pipelined DSPs in parallel. Each DSP provides only used for modular multiplication but also provides 448-bit additions and subtractions (using an internal carry-save representation).

 Eventually, data path expansion denotes the use of a full 448-bit data path for memory access as well as for arithmetic operations unlike other designs (cf. Table 2) in order to allow maximum parallelization and optimal performance within all sub-modules. In general, this allows to provide the FAU with all operands on time avoiding or reducing idling phases due to load or store operations.
and multiplications. Hence, sharing the DSPs allows implementing Carry-Save Additions and Subtractions as well as Schoolbook Multiplications with Interleaved Reduction within a single unit. The final accumulation and reduction of the results is performed by the RC.

Reduction Core (RC). The RC consists of another five DSPs, three to pre-add partial products and two to accumulate the carries and to perform the final reduction. In general, the reduction process is performed serially on 17-bit words in order to take advantage of the internal shift operation of the DSPs. However, due to the serial design, the reduction is the bottleneck of the Field Arithmetic Unit. Still, both, the AC and the RC, can be operated independently and in parallel in order to alleviate the disadvantage of the serial design.

5.2 Side-Channel Countermeasures

In this section we explain the necessary modifications of our architecture in order to provide a basic protection against side-channel attacks using point randomization and scalar blinding.

5.2.1 Point Randomization

As explained in Section 3.3.1, point randomization uses a randomly chosen parameter $\lambda$ which is applied to the base point. In general, this process initializes the $z$-coordinate with $\lambda$ and uses a modular multiplication in order to update the $x$-coordinate. Hence, this countermeasure can be integrated easily, mainly by using an additional multiplication call during the initialization phase of the implementation.

5.2.2 Scalar Blinding

According to Section 3.3.2, the scalar is blinded by a random multiple of the group order and due to the special structure of the underlying prime, it is required to use random factors of at least half of the field size [16]. Hence, the secret scalar is blinded and expanded by 221 bits using two additional DSP units. In particular, we opted to use a blinding factor of $221 = 13 \cdot 17$ bits, since it perfectly fits to the multiplier within the DSP unit that can perform $24 \times 17$-bit signed multiplications. However, based on the increased scalar size, the overall performance of the point multiplication is reduced due to additional Montgomery steps that have to be performed.

6. RESULTS

In this section, we present implementation and performance results after place-and-route (PAR) for both designs and provide a comparison to related work. Note, that all results were obtained for a Xilinx XC7Z020CLG484-3 FPGA using the Vivado Design Suite 2015.4.

6.1 Implementation Results

Table 1 provides the implementation results of both designs (including and excluding side-channel countermeasures) in terms of occupied resources. Obviously, the limiting factor for both designs (considering the maximum degree of parallelization for multi-core architectures) is the number of DSP units for the given platform. However, although the resource utilization slightly increases for the protected design, both variants theoretically allow to place up to six cores in parallel (on the chosen mid-range Xilinx Zynq FPGA device).

Table 1 also summarizes the performance results for both designs broken down to the group and field arithmetic operations. Since the unprotected design can achieve a higher maximum clock frequency (357 MHz) compared to the protected design (335 MHz), operations are slightly faster although the number of clock cycles are the same for the field arithmetic and group operations. However, due to the increased bit-size of the blinded scalar, the final point multiplication of the protected design involves additional iterations of the Montgomery ladder algorithm. This leads to an increase of the overall latency in terms of clock cycles by 45%.

6.2 Comparison

To the best of our knowledge, there are only few implementations of ECC schemes for FPGA devices publicly available that target a security level equal or above 128 bits. Along with the fact that modern FPGA architectures have evolved alot and now include more powerful features compared to earlier generation, a fair and meaningful discussion or comparison of different designs and implementations with previous works is not straightforward. Nevertheless, we like to put our results in the context with existing implementations to allow the reader a quick overview on other designs and architectures. Again, we like to emphasize that a simple comparison is not possible.

Table 2 lists several FPGA architectures of ECC schemes over prime fields, including standardized NIST primes (P-224 and P256), Curve25519 (second candidate of RFC 7748) and an implementation of an elliptic curve called FourQ, recently presented at CHES 2016. Note, however, that all these implementations target a lower level of security (112-128 bits) which implies a smaller field size and thus less arithmetic complexity for the basic operations. Still, we would like to highlight some observations from our design in comparison to previous works.

Given the fact, that time complexity is assumed to grow cubic in the field size, we can extrapolate a performance loss of 15-20% in comparison to the implementations listed in Table 2. In this regard, our implementation exceeds these expectations due to its manually optimized architecture, although, slightly more resources are occupied.

### Table 1: Resource utilization and performance results after place-and-route

<table>
<thead>
<tr>
<th>Aspect</th>
<th>Unprotected</th>
<th>SCA-Protected</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Resource Utilization</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LUTs</td>
<td>2555 (4.80%)</td>
<td>3563 (6.73%)</td>
</tr>
<tr>
<td>FFs</td>
<td>7049 (6.63%)</td>
<td>7423 (6.98%)</td>
</tr>
<tr>
<td>LSs</td>
<td>1580 (11.88%)</td>
<td>1648 (12.39%)</td>
</tr>
<tr>
<td>DSPs</td>
<td>33 (15.00%)</td>
<td>35 (15.91%)</td>
</tr>
<tr>
<td>BRAMs</td>
<td>14 (10.00%)</td>
<td>14 (10.00%)</td>
</tr>
<tr>
<td><strong>Performance</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Addition</td>
<td>11.20ns/4 cycles</td>
<td>11.92ns/4 cycles</td>
</tr>
<tr>
<td>Subtraction</td>
<td>11.20ns/4 cycles</td>
<td>11.20ns/4 cycles</td>
</tr>
<tr>
<td>Multi.</td>
<td>89.60ns/32 cycles</td>
<td>95.36ns/32 cycles</td>
</tr>
<tr>
<td>Reduction</td>
<td>180.80ns/36 cycles</td>
<td>187.20ns/36 cycles</td>
</tr>
<tr>
<td>Mont. Step</td>
<td>1.84µs/658 cycles</td>
<td>1.96µs/658 cycles</td>
</tr>
<tr>
<td>Inversion</td>
<td>92.3µs/3276 cycles</td>
<td>98.3µs/3276 cycles</td>
</tr>
<tr>
<td>Point Multi.</td>
<td>0.9ns/32826 cycles</td>
<td>1.4ns/473926 cycles</td>
</tr>
</tbody>
</table>
6.3 Curve25519 vs. Curve448 in Hardware

The underlying finite field and its corresponding base field arithmetic is an essential difference of Curve448 compared to Curve25519. Instead of a Pseudo Mersenne prime, Curve448 uses a Solinas prime which results in the need for a more complex reduction scheme. In combination with the significantly larger field size, this required a completely different modular multiplier design to still fully benefit from (additionally required) DSP units. In this context, the wider multiplier design of Curve448 was augmented to completely absorb the modular addition and subtraction operation in its data path, resulting in a single component for all modular operations. As a consequence, this single component design enabled a simple memory architecture on the higher arithmetic level compared to the more complex butterfly architecture of Curve25519 to connect two separate arithmetic components.

7. CONCLUSION

In this work we presented a new architecture for ECC based on Curve448, the elliptic curve for high-security application as defined in RFC 7748. Although Curve448 was mainly designed for efficient implementations on software platforms, we demonstrated that it also perfectly maps to modern FPGA architectures. Further, we showed that our implementation can be augmented with countermeasures against side-channel attacks at moderate costs. Whereas our unprotected allows more than 1000 point multiplications per second on a mid-range Xilinx XC7Z020 FPGA, our protected design achieves a performance of about 700 operations per second. Finally, we like to underline that the performance of both single core implementations should be sufficient to handle the required number of operations per second. Finally, we like to underline that the performance of both single core implementations should be sufficient to match the requirements a large number of high-security applications.

8. REFERENCES